Quick-Check Questions

Contents

1 Question Bank **2**
1.1 Statistics . . . 2
1.1.1 Definition . . . 2
1.1.2 Explain It in Your Own Words . . . 2
1.1.3 Derivation/Computation . . . 2
1.1.4 Provide the Formula . . . 2
1.2 Linear Algebra . . . 3
1.2.1 Definition . . . 3
1.2.2 Explain It in Your Own Words . . . 3
1.2.3 Derivation/Computation . . . 3
1.2.4 Provide the Formula . . . 4
1.3 Differential Equations . . . 4
1.3.1 Definition . . . 4
1.3.2 Explain It in Your Own Words . . . 4
1.3.3 Derivation/Computation . . . 4
1.3.4 Provide the Formula . . . 5

2 Classification on Three Axes **6**
2.1 Placement: Statistics . . . 7
2.2 Placement: Linear Algebra . . . 9
2.3 Placement: Differential Equations . . . 10

1 Question Bank

Each question is labeled **Topic-Type#**, with topics **S** (Statistics), **LA** (Linear Algebra), **DE** (Differential Equations) and types **D** (Definition), **E** (Explain It in Your Own Words), **C** (Derivation/Computation), **F** (Provide the Formula).

1.1 Statistics

1.1.1 Definition

S-D1 What is a discrete variable?

S-D2 What does the interquartile range (IQR) measure?

S-D3 What is a probability mass function?

S-D4 What is the standard error of a sample mean?

S-D5 What is a point estimate?

1.1.2 Explain It in Your Own Words

S-E1 In your own words, what does a z-score tell you about a data value?

S-E2 How does Chebyshev's theorem describe the spread of data around its mean?

S-E3 What does it mean for two events to be independent?

S-E4 What does the Central Limit Theorem say about the sampling distribution of sample means?

S-E5 What does a confidence interval communicate about an unknown population parameter?

1.1.3 Derivation/Computation

S-C1 Find the mean of the data set $2, 4, 6, 8$.

S-C2 A fair coin is flipped three times. What is the probability of getting exactly two heads?

S-C3 If X is binomial with $n = 5$ and $p = 0.2$, calculate $P(X = 1)$.

S-C4 A population has standard deviation 12, and a sample has size 36. Calculate the standard error of the sample mean.

S-C5 A value is 75, the population mean is 60, and the population standard deviation is 10. Calculate its z-score.

1.1.4 Provide the Formula

S-F1 Write the formula for the sample mean.

S-F2 Write the formula for the variance of a discrete random variable X using its probability mass function.

S-F3 Write Bayes' rule for calculating $P(A \mid B)$.

S-F4 Write the formula for the standard error of a sample proportion.

S-F5 Write the test statistic for a large-sample statistical test for a binomial population proportion.

1.2 Linear Algebra

1.2.1 Definition

LA-D1 What does it mean for a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ to be linearly independent?

LA-D2 What is the null space $\mathcal{N}(A)$ of an $m \times n$ matrix A?

LA-D3 What is a basis of a subspace S?

LA-D4 What are an eigenvalue and an eigenvector of a square matrix A?

LA-D5 What is an orthogonal matrix?

1.2.2 Explain It in Your Own Words

LA-E1 Why does a homogeneous system $A\mathbf{x} = \mathbf{0}$ with more unknowns than equations always have infinitely many solutions?

LA-E2 If A is a square matrix with $\det(A) = 0$, what can you say about A and about the system $A\mathbf{x} = \mathbf{b}$?

LA-E3 What does the Gram-Schmidt process do, and why is its output useful?

LA-E4 Explain geometrically what it means that $A = SDS^{-1}$, with D diagonal.

LA-E5 What does the Spectral Theorem guarantee for a real symmetric matrix A, and why is that convenient?

1.2.3 Derivation/Computation

LA-C1 Find the reduced row echelon form and the rank of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}.$$

LA-C2 Compute

$$\det\left(\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}\right).$$

LA-C3 Find the eigenvalues and one eigenvector for each eigenvalue of

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}.$$

LA-C4 Find the orthogonal projection of $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ onto the line spanned by $\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

LA-C5 Sketch the image of the unit square $[0, 1] \times [0, 1]$ under $T(\mathbf{x}) = A\mathbf{x}$ with

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix},$$

and label its vertices.

1.2.4 Provide the Formula

LA-F1 Write the formula for the inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, including when it exists.

LA-F2 Write the formula for the orthogonal projection of $\mathbf{b}$ onto the line spanned by a nonzero vector $\mathbf{a}$.

LA-F3 State the Rank-Nullity Theorem for an $m \times n$ matrix A.

LA-F4 Write the equation whose roots are the eigenvalues of a square matrix A.

LA-F5 Write the k-th step of the Gram-Schmidt process, which builds $\mathbf{v}_k$ from $\mathbf{x}_k$ and the already computed $\mathbf{v}_1, \ldots, \mathbf{v}_{k-1}$.

1.3 Differential Equations

1.3.1 Definition

DE-D1 What is the order of a differential equation?

DE-D2 When is an n^{th} order differential equation linear?

DE-D3 What is an initial value problem (IVP) for an n^{th} order differential equation?

DE-D4 For a linear equation $L[y] = b(x)$, what is the complementary solution y_c?

DE-D5 What is the Laplace transform of a function $f(t)$?

1.3.2 Explain It in Your Own Words

DE-E1 Explain why the set of solutions to a linear homogeneous differential equation $L[y] = 0$ is a subspace.

DE-E2 Explain why every solution of $L[y] = b(x)$ has the form $y = y_c + y_p$.

DE-E3 How does Euler's method approximate the solution of $y' = f(x, y)$, $y(x_0) = y_0$?

DE-E4 What does the Existence and Uniqueness Theorem guarantee for $y' = f(x, y)$, $y(x_0) = y_0$, and under which hypotheses?

DE-E5 Why is the Laplace transform useful for solving linear constant-coefficient initial value problems?

1.3.3 Derivation/Computation

DE-C1 Solve the initial value problem $y' = xy$, $y(0) = 2$.

DE-C2 Find the general solution of $y'' + 2y' + 5y = 0$.

DE-C3 Find a particular solution of $y'' + y = 3e^{2x}$ using undetermined coefficients.

DE-C4 Use two steps of Euler's method with $h = 0.1$ to approximate $y(0.2)$ for $y' = x + y$, $y(0) = 1$.

DE-C5 Sketch the slope field of $y' = y - x$ at the nine points (x, y) with $x, y \in \{-1, 0, 1\}$.

1.3.4 Provide the Formula

DE-F1 Write an integrating factor $m(x)$ and the general solution of $y' + p(x)y = q(x)$.

DE-F2 Write the update rule of Euler's method for $y' = f(x, y)$ with step h.

DE-F3 Write the Wronskian $W(x)$ of two differentiable functions $f_1(x)$ and $f_2(x)$.

DE-F4 Write $\mathcal{L}\{y''(t)\}$ in terms of $Y(s) = \mathcal{L}\{y(t)\}$ and the initial values.

DE-F5 Write the solution of $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \mathbf{x}_0$, and the series that defines e^{At}.

2 Classification on Three Axes

A student writes an answer by hand, takes a picture and uploads it. An AI model then transcribes the picture and reviews the transcription. We place every question at a point $(T, R, E) \in [0, 5]^3$, where larger values mean harder.

Definition 2.1 (Transcription difficulty T)
How hard it is for the AI to turn a photo of a handwritten answer into a faithful text or LaTeX version.

Definition 2.2 (Review difficulty R)
How hard it is for the AI, given a faithful transcription, to decide whether the answer is correct and to give useful feedback.

Definition 2.3 (Execution difficulty E)
How hard it is for a student who has followed the lectures to produce a correct answer in the few minutes available.

The three anchors below fix the scale; intermediate values interpolate between them.

	Transcription T	**Review** R	**Execution** E
1	A short sentence or a single number.	One correct value; an exact check.	Recall of a single fact.
3	Several lines mixing words and notation (fractions, sums, subscripts).	Several acceptable forms or wordings; key ingredients are checkable.	One short computation or a careful explanation.
5	A hand-drawn figure whose content is read spatially (sketch, slope field).	Open-ended; correctness depends on reasoning and needs judgment.	Several steps and concepts; slips are likely.

Matrices and row-reduction chains sit near $T = 4$: the content is exact, but its meaning depends on the alignment of entries. Such questions are hard to transcribe and easy to review, while explanations are the reverse.

Remark 1
- *Computation questions lie along $R \approx 1$: a single checkable answer, with T set by how the work is written (numbers, matrices, figures).*
- *Explain questions lie along $T \approx 2$ to 2.5 with $R \geq 3$: handwriting of prose is easy to read, grading it is not.*

- *The region with both T and R large is empty. A sketch that is also open-ended (e.g. "draw and explain") would land there.*

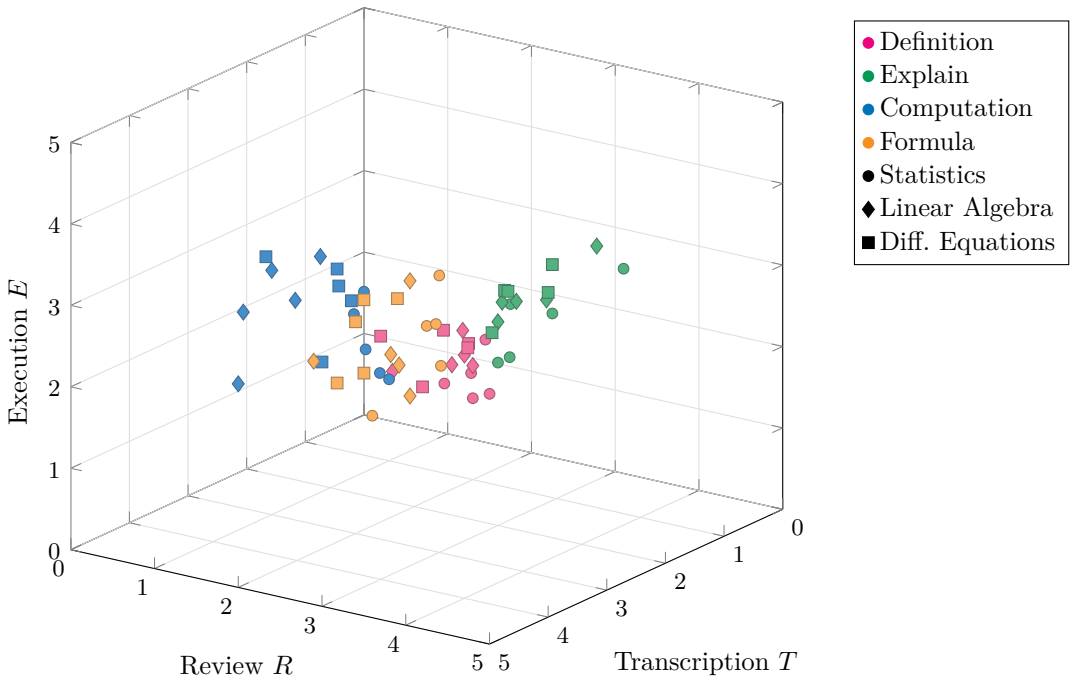


Figure 1: All questions in $[0, 5]^3$. Color gives the type, the marker gives the topic.

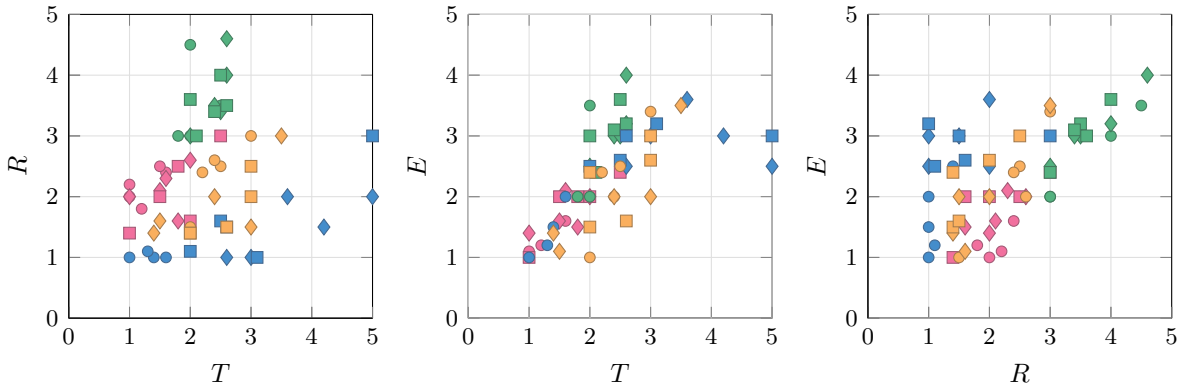


Figure 2: Projections onto the (T, R), (T, E) and (R, E) planes.

2.1 Placement: Statistics

S-D1 $(T, R, E) = (1, 2, 1)$. *Transcription:* One handwritten sentence with no notation. *Review:* Wording varies, but the check reduces to one idea (countably many values). *Execution:* Recall of a single fact.

S-D2 $(T, R, E) = (1.2, 1.8, 1.2)$. *Transcription:* A sentence, possibly with $Q_3 - Q_1$. *Review:* Two checkable ingredients (middle half, $Q_3 - Q_1$); little room for interpretation. *Execution:* Recall of a

single fact.

S-D3 $(T, R, E) = (1.6, 2.4, 1.6)$. *Transcription:* A sentence mixed with light notation ($P(X = x)$, a sum). *Review:* The reviewer must decide whether omitting the two properties or the word discrete is acceptable. *Execution:* Recall, but students often confuse it with a density.

S-D4 $(T, R, E) = (1.5, 2.5, 2)$. *Transcription:* A sentence with an optional fraction and root. *Review:* A correct answer may be verbal or the formula; answers that describe the population standard deviation must be caught. *Execution:* Requires separating the sample, population and sampling distribution.

S-D5 $(T, R, E) = (1, 2.2, 1.1)$. *Transcription:* One sentence with no notation. *Review:* Wording varies; the check is single number from a sample, estimating a parameter. *Execution:* Recall of a single fact.

S-E1 $(T, R, E) = (2, 3, 2)$. *Transcription:* A few handwritten sentences. *Review:* Open wording; the reviewer must check both distance in standard deviations and the meaning of the sign. *Execution:* A familiar idea that needs one clear sentence.

S-E2 $(T, R, E) = (2.5, 3.5, 3)$. *Transcription:* Sentences mixed with the bound $1 - 1/k^2$. *Review:* Vague answers ("most data is near the mean") need a judgment on partial credit; "any distribution" is easy to miss. *Execution:* Requires the quantitative statement, not only the intuition.

S-E3 $(T, R, E) = (1.8, 3, 2)$. *Transcription:* Sentences with optional probability notation. *Review:* Several correct forms; confusion with disjoint events must be detected. *Execution:* Familiar, one idea.

S-E4 $(T, R, E) = (2.5, 4, 3)$. *Transcription:* Several sentences with some notation. *Review:* Three components (normality, mean, spread) and the common misstatement about the data itself becoming normal. *Execution:* Requires a precise statement of a subtle theorem.

S-E5 $(T, R, E) = (2, 4.5, 3.5)$. *Transcription:* A few handwritten sentences. *Review:* The standard misinterpretation (probability that this interval contains μ) is subtle to grade and to explain back. *Execution:* Correct interpretation is a known stumbling block.

S-C1 $(T, R, E) = (1, 1, 1)$. *Transcription:* A single number. *Review:* Exact check against 5. *Execution:* One line of arithmetic.

S-C2 $(T, R, E) = (1.6, 1, 2)$. *Transcription:* A fraction, possibly with a list of outcomes. *Review:* Exact check against 3/8. *Execution:* Counting outcomes, short.

S-C3 $(T, R, E) = (2, 1.4, 2.5)$. *Transcription:* A binomial coefficient and powers. *Review:* A single value, with rounding tolerance to set. *Execution:* Formula recall plus a power computation by hand.

S-C4 $(T, R, E) = (1.4, 1, 1.5)$. *Transcription:* A fraction with a root. *Review:* Exact check against 2. *Execution:* One formula, one division.

S-C5 $(T, R, E) = (1.3, 1.1, 1.2)$. *Transcription:* A short fraction. *Review:* Exact check against 1.5. *Execution:* One formula, one division.

S-F1 $(T, R, E) = (2, 1.5, 1)$. *Transcription:* A summation with indices. *Review:* Essentially one form. *Execution:* Recall.

S-F2 $(T, R, E) = (2.5, 2.5, 2.5)$. *Transcription:* A sum with squares and subscripts. *Review:* Two standard equivalent forms must both be accepted. *Execution:* Recall, often mixed up with the sample variance.

S-F3 $(T, R, E) = (2.4, 2.6, 2)$. *Transcription:* A fraction of conditional probabilities. *Review:* Expanded and compact denominators are both correct. *Execution:* Recall.

S-F4 $(T, R, E) = (2.2, 2.4, 2.4)$. *Transcription:* A root over a fraction with hats. *Review:* Versions with p or $\hat{p}$ both appear in practice. *Execution:* Recall.

S-F5 $(T, R, E) = (3, 3, 3.4)$. *Transcription:* Nested fraction, root, hats and subscripts. *Review:* The reviewer must check that p_0, not $\hat{p}$, is in the denominator. *Execution:* Recall with a detail students often get wrong.

2.2 Placement: Linear Algebra

LA-D1 $(T, R, E) = (2, 2.6, 2)$. *Transcription:* A linear combination with subscripts. *Review:* The answer "no vector is a multiple of another" is only correct for two vectors and must be caught. *Execution:* Precise recall of the definition.

LA-D2 $(T, R, E) = (1.8, 1.6, 1.5)$. *Transcription:* Set-builder notation. *Review:* One form; the check is $A\mathbf{x} = \mathbf{0}$ and $\mathbb{R}^n$ (not $\mathbb{R}^m$). *Execution:* Recall.

LA-D3 $(T, R, E) = (1, 2, 1.4)$. *Transcription:* One sentence. *Review:* Two ingredients (independent, spanning) to check. *Execution:* Recall.

LA-D4 $(T, R, E) = (1.5, 2.1, 1.6)$. *Transcription:* A short equation with a sentence. *Review:* The condition $\mathbf{v} \neq \mathbf{0}$ is the usual omission. *Execution:* Recall.

LA-D5 $(T, R, E) = (1.6, 2.3, 2.1)$. *Transcription:* A sentence with a transpose. *Review:* Several equivalent forms ($Q^{-1} = Q^{\mathsf{T}}$, orthonormal columns); "orthogonal columns" is not enough. *Execution:* Recall, with the orthogonal versus orthonormal trap.

LA-E1 $(T, R, E) = (2.4, 3.5, 3)$. *Transcription:* Sentences with light notation. *Review:* Two separate reasons are needed (consistency, a free variable); arguments vary in form. *Execution:* A short argument, not a recalled fact.

LA-E2 $(T, R, E) = (2, 3, 2.5)$. *Transcription:* Sentences. *Review:* Many correct equivalent statements; "no solution" alone is incomplete. *Execution:* Connecting two facts.

LA-E3 $(T, R, E) = (2.6, 4, 3.2)$. *Transcription:* Several sentences, possibly with a projection formula. *Review:* Open-ended usefulness part; many acceptable reasons. *Execution:* Requires a summary of a procedure and its purpose.

LA-E4 $(T, R, E) = (2.6, 4.6, 4)$. *Transcription:* Sentences, possibly a small sketch. *Review:* Geometric explanations vary widely and need judgment. *Execution:* Requires a conceptual picture, not a procedure.

LA-E5 $(T, R, E) = (2.5, 3.4, 3)$. *Transcription:* Sentences with a factorization. *Review:* The guarantee is checkable; the convenience part is open. *Execution:* Recall of a theorem with an interpretation.

LA-C1 $(T, R, E) = (4.2, 1.5, 3)$. *Transcription:* A chain of matrices with row-operation labels; column alignment carries the meaning. *Review:* The RREF is unique, so the final matrix and rank are exact checks. *Execution:* A few row operations where slips happen.

LA-C2 $(T, R, E) = (3, 1, 3)$. *Transcription:* A cofactor expansion with 2×2 determinants. *Review:* Exact check against 8. *Execution:* A short expansion with sign bookkeeping.

LA-C3 $(T, R, E) = (3.6, 2, 3.6)$. *Transcription:* A characteristic polynomial, two small matrices and column vectors. *Review:* Eigenvalues are exact; eigenvectors are checked by $A\mathbf{v} = \lambda\mathbf{v}$ since any multiple is correct. *Execution:* Factor a quadratic, then solve two singular systems.

LA-C4 $(T, R, E) = (2.6, 1, 2.5)$. *Transcription:* Column vectors and dot products. *Review:* Exact check against one vector. *Execution:* Formula recall plus two dot products.

LA-C5 $(T, R, E) = (5, 2, 2.5)$. *Transcription:* A hand-drawn figure; the AI must read shape, axes and labels spatially. *Review:* Once read, four vertices are exact checks. *Execution:* Four matrix-vector products and a sketch.

LA-F1 $(T, R, E) = (3, 1.5, 2)$. *Transcription:* A matrix with signs and a scalar fraction. *Review:* One form; the check is entry by entry. *Execution:* Recall, sign placement is the common error.

LA-F2 $(T, R, E) = (2.4, 2, 2)$. *Transcription:* A fraction of dot products. *Review:* Equivalent forms with $\|\mathbf{a}\|^2$ or $\mathbf{a}^\mathsf{T}\mathbf{b}$. *Execution:* Recall.

LA-F3 $(T, R, E) = (1.4, 1.4, 1.4)$. *Transcription:* One short equation. *Review:* One form; the check is n versus m. *Execution:* Recall.

LA-F4 $(T, R, E) = (1.5, 1.6, 1.1)$. *Transcription:* One short equation. *Review:* $\det(\lambda I - A) = 0$ is equally correct. *Execution:* Recall.

LA-F5 $(T, R, E) = (3.5, 3, 3.5)$. *Transcription:* A sum of fractions with several subscripts. *Review:* Index ranges and $\mathbf{x}_j$ versus $\mathbf{v}_j$ must be checked; normalized variants exist. *Execution:* Recall of a general formula, not only the two-vector case.

2.3 Placement: Differential Equations

DE-D1 $(T, R, E) = (1, 1.4, 1)$. *Transcription:* One sentence. *Review:* One idea, little room for interpretation. *Execution:* Recall.

DE-D2 $(T, R, E) = (2.5, 3, 2.4)$. *Transcription:* A general form with subscripts and derivative orders. *Review:* Verbal answers ("no products or powers of y") must be judged against the general form. *Execution:* Recall of a precise form.

DE-D3 $(T, R, E) = (1.8, 2.5, 2)$. *Transcription:* A sentence with some notation. *Review:* The single point and the number of conditions separate it from a boundary value problem. *Execution:* Recall.

DE-D4 $(T, R, E) = (1.5, 2, 2)$. *Transcription:* A sentence with short notation. *Review:* One idea to check. *Execution:* Recall, sometimes confused with y_p.

DE-D5 $(T, R, E) = (2, 1.6, 2)$. *Transcription:* An improper integral. *Review:* One standard form. *Execution:* Recall.

DE-E1 $(T, R, E) = (2.6, 3.5, 3.2)$. *Transcription:* Sentences with a short computation. *Review:* The argument can be organized several ways; the reviewer must check that linearity is actually used. *Execution:* A short proof.

DE-E2 $(T, R, E) = (2.5, 4, 3.6)$. *Transcription:* Sentences with a short computation. *Review:* Explanations range from the clean argument to vague restatements; judgment needed. *Execution:* Requires the subtraction idea, not recall.

DE-E3 $(T, R, E) = (2.1, 3, 2.4)$. *Transcription:* Sentences, perhaps a small sketch. *Review:* Wording varies; the check is tangent-line steps with slope given by f. *Execution:* Description of a familiar procedure.

DE-E4 $(T, R, E) = (2.4, 3.4, 3.1)$. *Transcription:* Sentences with a partial derivative. *Review:* Both hypotheses and both conclusions must be present; paraphrases vary. *Execution:* Precise recall of a theorem.

DE-E5 $(T, R, E) = (2, 3.6, 3)$. *Transcription:* A few sentences. *Review:* Several valid reasons; the reviewer must judge completeness. *Execution:* Requires a view of the whole method.

DE-C1 $(T, R, E) = (2.6, 1.5, 3)$. *Transcription:* Separation of variables with integrals and logarithms. *Review:* One answer, checkable by substitution; equivalent forms are easy to compare. *Execution:* Separate, integrate, apply the initial condition.

DE-C2 $(T, R, E) = (2.5, 1.6, 2.6)$. *Transcription:* A quadratic formula and exponentials with trigonometric functions. *Review:* One answer up to naming of constants. *Execution:* Roots of a quadratic with complex roots, then the basis rule.

DE-C3 $(T, R, E) = (2, 1.1, 2.5)$. *Transcription:* A few short lines. *Review:* Exact check against $\frac{3}{5}e^{2x}$. *Execution:* Guess, differentiate, match one coefficient.

DE-C4 $(T, R, E) = (3.1, 1, 3.2)$. *Transcription:* A small table of x_n, y_n and slopes. *Review:* Exact check against 1.22 (and 1.1). *Execution:* Two iterations of arithmetic with decimals.

DE-C5 $(T, R, E) = (5, 3, 3)$. *Transcription:* A hand-drawn grid of short segments; the AI must estimate their angles. *Review:* Segment slopes are judged with tolerance from a drawing. *Execution:* Nine slope evaluations and a careful sketch.

DE-F1 $(T, R, E) = (3, 2.5, 3)$. *Transcription:* Exponentials of integrals, nested. *Review:* The position of C is the key check; equivalent forms exist. *Execution:* Recall of two linked formulas.

DE-F2 $(T, R, E) = (2, 1.4, 1.5)$. *Transcription:* Short equations with subscripts. *Review:* One form. *Execution:* Recall.

DE-F3 $(T, R, E) = (2.6, 1.5, 1.6)$. *Transcription:* A 2×2 determinant with primes. *Review:* Matrix and expanded forms, both easy to check. *Execution:* Recall.

DE-F4 $(T, R, E) = (2, 1.4, 2.4)$. *Transcription:* One short line. *Review:* One form. *Execution:* Recall where signs and powers of s slip.

DE-F5 $(T, R, E) = (3, 2, 2.6)$. *Transcription:* A series with matrix powers and factorials. *Review:* Sum or expanded form, both easy to compare. *Execution:* Recall of two linked facts.